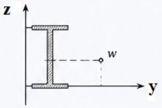
BACKGROUND EQUATIONS – SUMMARY

1. ELASTIC BEHAVIOR

1.1 Normal stresses (elastic) – different materials E_i

$$\sigma = \frac{E}{E_{ref}} \left[\frac{N}{A} + \frac{(M_y I_z + M_z I_{yz})}{(I_y I_z - I_{yz}^2)} z - \frac{(M_z I_y + M_y I_{yz})}{(I_y I_z - I_{yz}^2)} y + \frac{B}{I_w} w \right]$$

E = Young's modulus of the fibre; E_{ref} = Young's modulus reference material

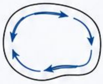


1.2 Shear stresses (elastic) – thin walled sections

$$\tau_t = \tau_{t0} + \left[\iint \frac{E}{E_{ref}} z dA \frac{(V_z I_z - V_y I_{yz})}{(I_y I_z - I_{yz}^2)} + \iint \frac{E}{E_{ref}} y dA \frac{(V_y I_y - V_z I_{yz})}{(I_y I_z - I_{yz}^2)} + \iint \frac{E}{E_{ref}} w dA \frac{(T_w)}{I_w} \right]$$

1.3 Closed thin-walled sections

At each independent closed cell:



$$\oint \frac{\tau}{G} ds = 2 A_{cell} \frac{d\theta_x}{dx}$$

$$\oiint d\tau \, dA = M_x$$

(warping)

$$w_{i+1} - w_i = \left(\rho(i) - \frac{q(i)}{G(i) t(i) \theta_x'} \right) L(i)$$

Solved with Minimum norm least-squares solution to linear equations

2. PLASTIC BEHAVIOR – 2 METHODS

2.1 Method 1 – To obtain interaction diagrams N–M_y–M_z & M_y–M_z–B

For each combination of actions, the plastification factor ξ is obtained.

Case 1: Without considering Bimoment as a constraint

Maximize ξ subject to:

$$N = \sum_{i=1}^{n} A(i) \sigma(i) = \xi N_1$$

$$M_y = \sum_{i=1}^{n} z(i) A(i) \sigma(i) = \xi M_{y1}$$

$$M_z = -\sum_{i=1}^{n} y(i) A(i) \sigma(i) = \xi M_{z1}$$

Case 2: Considering Bimoment as a constraint

Maximize ξ subject to:

$$N = \sum_{i=1}^{n} A(i) \sigma(i) = \xi N_1$$

$$M_y = \sum_{i=1}^{n} z(i) A(i) \sigma(i) = \xi M_{y1}$$

$$M_z = -\sum_{i=1}^{n} y(i) A(i) \sigma(i) = \xi M_{z1}$$

$$B = \sum_{i=1}^{n} \omega(i) A(i) \sigma(i) = \xi B_1$$

Material stress limits

Normal stress only:

$$f_c < \sigma(i) < f_t$$

Von Mises stress (priority shear*)

$$-\sqrt{f_y^2 - 3\tau^2} < \sigma(i) < \sqrt{f_y^2 - 3\tau^2}$$

* First shear stresses computed due to V_y, V_z, T_t, T_w (elastic behavior).

2.2 Method 2 – Plastic VML (Von Mises Linearized)

To obtain interaction diagram M_y–V_z–N, the plastification factor ξ is obtained for each combination.

Case 2: Considering Bimoment as a constraint

Maximize ξ subject to:

$$N = \sum_{i=1}^{n} A(i) \sigma(i) = \xi N_1$$

$$M_y = \sum_{i=1}^{n} z(i) A(i) \sigma(i) = \xi M_{y1}$$

$$M_z = -\sum_{i=1}^{n} y(i) A(i) \sigma(i) = \xi M_{z1}$$

$$B = \sum_{i=1}^{n} \omega(i) A(i) \sigma(i) = \xi B_1$$

$$V_y = \sum_{i=1}^{n} \cos\alpha(i) A(i) \tau(i) = \xi V_{y1}$$

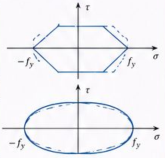
$$V_z = \sum_{i=1}^{n} \sin\alpha(i) A(i) \tau(i) = \xi V_{z1}$$

$$M_x = \sum_{i=1}^{n} d(i) A(i) \tau(i) = \xi M_{x1}$$

Linearized Von Mises: 2 methods

Method 1 (implemented in this software)

Method 2 (user can implement using source code in MATLAB)

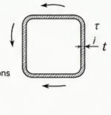


—— Yield surface

- - - Linearized approximation

SYMBOLS – MAIN MEANING

N	Axial force
M_y, M_z	Bending moments about y and z
B	Bimoment (warping moment)
V_y, V_z	Shear forces
T_t, T_w	Torsional moments (St. Venant and warping)
M_x	Torque due to shear flow
$A(i)$	Area of element i
$\sigma(i), \tau(i)$	Normal and shear stress in element i
y, z	Coordinates
w	Sectorial coordinate (warping)
ω	Warping function
I_y, I_z	Second moments of area
I_{yz}	Product of inertia
I_w	Warping constant
G	Shear modulus
t	Thickness
τ_t	Shear flow
θ_x	Angle of twist per unit length
A_{cell}	Area of closed cell
$\rho(i), q(i)$	Geometric and shear flow functions
$L(i)$	Length of element i
ξ	Plastification factor



KEY IDEA: Compute internal forces from stresses (elastic), then find the maximum multiplier ξ that keeps stresses within material limits (plastic). This builds the interaction (capacity) diagrams.

SUMMARY PDF 2 – DYNAMIC ANALYSIS

1. DAMPING MATRIX $\boldsymbol{C}$

Option 0

$$\boldsymbol{C} = 2 \cdot \varepsilon \cdot \omega_1 \cdot \boldsymbol{M}$$

Option 1

$$\boldsymbol{C} = 2 \cdot \varepsilon \, \frac{\omega_1 \cdot \omega_2}{\omega_1 + \omega_2} \, \boldsymbol{M} + \frac{2 \cdot \varepsilon}{\omega_1 \cdot \omega_2} \, \boldsymbol{K}$$

Option 2

$$\boldsymbol{C} = \boldsymbol{a_0} \cdot \boldsymbol{M} + \boldsymbol{a_1} \cdot \boldsymbol{K}$$

Where:

ε : Damping ratio (dimensionless)

ω_1, ω_2 : Natural frequencies (rad/s)

$\boldsymbol{M}$: Mass matrix

$\boldsymbol{K}$: Stiffness matrix

$\boldsymbol{C}$: Damping matrix

2. SOLVING DYNAMIC PROBLEMS – NEWMARK ALGORITHM

Newmark parameters: $\{\beta, \gamma\}$

2.1. Effective matrix and effective load vector

$$\boldsymbol{K}_1 = \boldsymbol{K} + \frac{\boldsymbol{M}}{\beta \, \Delta t^2} + \frac{\gamma \, \boldsymbol{C}}{\beta \, \Delta t}$$

$$\boldsymbol{P}_2 = \boldsymbol{F}_2 + \boldsymbol{M}\left(\frac{d_1}{\beta \, \Delta t^2} + \frac{v_1}{\beta \, \Delta t} + \left(\frac{1}{2\beta} - 1\right) a_1\right) + \boldsymbol{C}\left(\frac{\gamma d_1}{\beta \, \Delta t} + \left(\frac{\gamma}{\beta} - 1\right) v_1 + \left(\frac{\gamma}{2\beta} - 1\right) \Delta t \, a_1\right)$$

2.2. Compute new displacements, velocities and accelerations

$$\boldsymbol{K}_1 \, \boldsymbol{d}_2 = \boldsymbol{P}_2$$

$$a_2 = \frac{1}{\beta \, \Delta t^2}\left(d_2 - d_1 - v_1 \, \Delta t\right) - \left(\frac{1}{2\beta} - 1\right) a_1$$

$$v_2 = \frac{\gamma}{\beta \, \Delta t}\left(d_2 - d_1\right) + \left(1 - \frac{\gamma}{\beta}\right) v_1 + \left(1 - \frac{\gamma}{2\beta}\right) \Delta t \, a_1$$

Where:

Δt : Time increment

d_1, v_1, a_1 : Displacement, velocity and acceleration at time t_1

d_2, v_2, a_2 : Displacement, velocity and acceleration at time $t_2 = t_1 + \Delta t$

F_2 : External force vector at time t_2

SUMMARY PDF 1 – COMPOSITE MATERIALS CONSTITUTIVE EQUATIONS

1. PLANE STRESS ELASTICITY RELATIONS FOR COMPOSITE MATERIALS

For a composiive lamina under plane stress ($\sigma_3 = \tau_{13} = \tau_{23} = 0$), the constitutive relations between stresses and strains can be written in matrix form as follows:

$$\sigma = [Q]\,\varepsilon$$

where:

$$[\sigma] = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \quad [\varepsilon] = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$[Q]$ is the reduced stiffness matrix (3×3), defined in the principal material axes (1–2 system).

1.1. In the 1–2 coordinate system (aligned with material directions):

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

where:

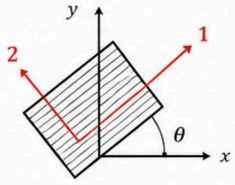
$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \qquad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \qquad Q_{66} = G_{12}$$

2. TRANSFORMED STIFFNESS MATRIX FOR A ROTATED LAMINA

If the material axes (1–2) are rotated by an angle θ with respect to the global axes $(x-y)$, the stiffness matrix must be transformed. Let $m = \cos(\theta)$ and $n = \sin(\theta)$.
The transformed stiffness matrix $[\bar{Q}]$ is given by:

- $\bar{Q}_{11} = Q_{11}\, m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}\, n^4$
- $\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}\,(m^4 + n^4)$
- $\bar{Q}_{22} = Q_{11}\, n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}\, m^4$
- $\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})m^3n + (Q_{12} - Q_{22} + 2Q_{66})mn^3$
- $\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$
- $\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})m^2n^2 + Q_{66}\,(m^4 + n^4)$



Thus, the transformed matrix is:

$$[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}$$

- E_1, E_2 : Young's moduli in the 1 and 2 directions
- G_{12} : Shear modulus in the 1–2 plane
- ν_{12}, ν_{21} : Poisson's ratios (with $\nu_{12}E_2 = \nu_{21}E_1$)
- θ : Angle of rotation between axes (1–2) and $(x-y)$
- $m = \cos(\theta), \quad n = \sin(\theta)$