BACKGROUND EQUATIONS

Matrix method equilibrium equations:

$$[K_L]\{d\} \leftarrow \frac{1}{2}\int_0^L \left(EA\left(\frac{da}{dx}\right)^2 + GJ\left(\frac{d\theta_x}{dx}\right)^2 + EI_y\left(\frac{d^2w}{dx^2}\right)^2 + EI_y\left(\frac{d^3v}{dx^2}\right)^2 + EI_w\left(\frac{d^2\theta_x}{dx^2}\right)^2\right)dx$$

$$[K_G]\{d\} \leftarrow \frac{1}{2}\int_0^L \left(N\left[\left(\frac{dv}{dx}\right)^2 + \left(\frac{dw}{dx}\right)^2 + 2z_{sc}\left(\frac{dv}{dx}\right)\left(\frac{d\theta_z}{dx}\right) - 2y_{sc}\left(\frac{dw}{dx}\right)\left(\frac{d\theta_x}{dx}\right) + \left(\frac{d\theta_x}{dx}\right)^2 r_o^2\right]\right.$$
$$+ 2M_6\left(\frac{d^2w}{dx^3}\right)\left(\frac{d\theta_x}{dx}\right) + 2M_y\left(\frac{d^2v}{dx^3}\right)\left(\frac{d\theta_x}{dx}\right)$$
$$\left. + (M_9\beta_z - M_B\beta_y + B\beta_w)\left(\frac{d\ \theta_x}{dx}\right)^2\right)dx$$

$$\beta_z = \frac{1}{I_y}\iint z(y^2+z^2)\,dA - 2z_{sc} \qquad \beta_y = \frac{1}{I_z}\iint y(y^2+z^2)\,dA - 2y_{sc} \qquad \beta_w = \frac{1}{I_w}\iint w(y^2+z^2)\,dA$$

$$\boxed{[K_L] + [K_G]\{d\} = \{f_{ext}\} + [K_G]\{\eta_{init}\}}$$

ℹ Take into account load position F_z, q_z (z position referred to shear center).

ELASTIC BEHAVIOR

Normal stresses elastic (different materials E_I):

$$\sigma = \frac{E}{E_{ref}}\left[\frac{N}{A} + \frac{(M_y I_y + M_y I_{yz})}{(I_y I_z - I_{yz}^3)} z - \frac{(M_y I_y + M_y I_{yz})}{(I_y I_z - I_{yz}^2)} y + \frac{B}{I_w} w\right]$$

$$ro^2 = \frac{Iy+Iz}{A} + y_{sc}^2 + z_{sc}^2$$

E young's modulus of the fibre : E_{ref} Young's modulus reference material

Elastic shear stresses, in thin walled sections:

$$\tau_t = \tau_{10} + \left[\iint \frac{E}{E_{ref}} z\,dA \frac{(V_z I_z - V_y I_{ys})}{(I_y I_z - I_{yz}^2)} + \iint \frac{E}{E_{ref}} y\,dA \frac{(V_y I_y - V_z I_{ys})}{(I_y I_z - I_{yz}^2)}\right.$$
$$\left. + \iint \frac{E}{E_{ref}} w\,dA \frac{(T_w)}{I_w}\right]$$

In closed thin walled at each independent closed cell:

$$\oint \frac{\tau}{G} ds = 2A_{cell}\frac{d\theta_x}{dx}$$

PLASTIC BEHAVIOR

To obtain the interaction diagram $N\ M_y\ M_z$ the plastification factor ξ can be obtained for each combination :

Case 1: Without considering bimoment as a constraint	Case 2: Considering bimoment constraint
Maximize ξ subject to:	Maximize ξ subject to:
$N = \sum_{i=1}^{n} A(i)\sigma(i) = \xi N_1$	$N = \sum_{i=1}^{n} A(i)\sigma(i) = \xi N_1$
$M_y = \sum_{i=1}^{n} z(i)A(i)\sigma(i) = \xi M_{y1}$	$M_y = \sum_{i=1}^{n} z(i)A(i)\sigma(i) = \xi M_{y1}$
$M_z = -\sum_{i=1}^{n} y(i)A(i)\sigma(i) = \xi M_{21}$	$M_z = -\sum_{i=1}^{n} y(i)A(i)\sigma(i) = \xi M_{21}$
	$B = \sum_{i=1}^{n} \omega(i)A(i)\sigma(i) = \xi B_1$

Normal stress only:

$$\boxed{f_c < \sigma(i) < f_t}$$

Von Mises stress (priority shear*)

$$\boxed{-\sqrt{f_y^3 - 3r^2} < \sigma(i) < \sqrt{f_y^3 - 3r^2}}$$

*First shear streses computed due to V_y, V_s, T_T, T_t, elastic behavior.