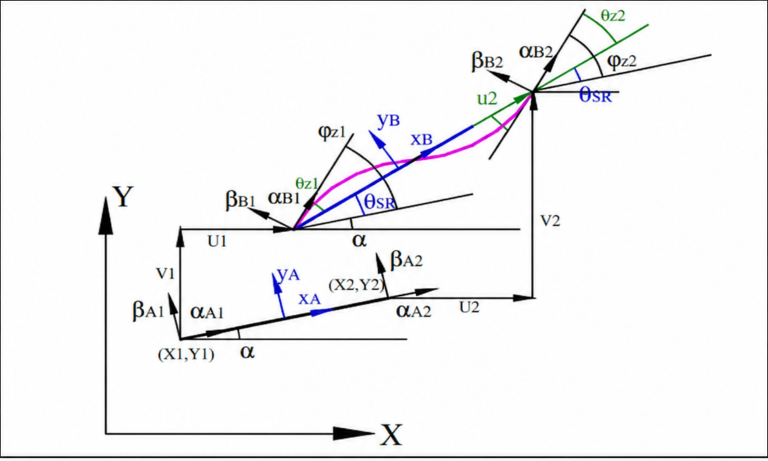
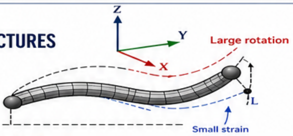


COROTATIONAL FORMULATION FOR GEOMETRIC NONLINEARITY IN BAR STRUCTURES

Large rotations. Small strains. Exact equilibrium.

When structures move beyond the comfort zone of linearity, intuition breaks. Rotations grow large, stiffness evolves, and classical formulations start whispering lies.

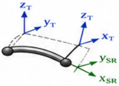
The corotational approach cuts through that noise with a simple but powerful idea: **separate motion into rigid-body rotation and pure deformation.**



1 KINEMATIC DECOMPOSITION

$$[R_T] = [R_D] \cdot [R_{SR}]$$

- $[R_T]$: total rotation (initial → deformed)
- $[R_{SR}]$: rigid-body rotation (element frame update)
- $[R_D]$: pure deformation (small, linearizable)



At each node:

$$[R_{D_i}] = [R_{T_i}] \cdot [R_{SR_i}]^T$$

For small rotations:

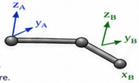
$$[R_D] \approx I + [W]$$

$$[W] = \begin{bmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{bmatrix}$$

We move the complexity into rotation, keep deformation small and linear.

2 ELEMENT FRAME UPDATE

$$[T_B] = [R_{SR}] \cdot [T_A]$$
$$\Rightarrow [R_{SR}] = [T_B] \cdot [T_A]^T$$

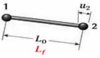


The element carries its own moving frame—
like a compass that rotates with the structure.

3 AXIAL DEFORMATION (PURE STRETCH)

$$u_2 = L_f - L_0$$

$$L_f = \sqrt{(X_2+u_2-X_1-U_1)^2 + (Y_2+V_2-Y_1-V_1)^2 + (Z_2+W_2-Z_1-W_1)^2}$$



No approximations.
Geometry speaks exactly.

4 PURE DEFORMATION VECTOR

$$\{d_D\} = \{\theta_{y1},\ \theta_{z1},\ \phi_1,\ u_2,\ \theta_{xD},\ \theta_{y2},\ \theta_{z2},\ \phi_2\}$$

$$\theta_{xD} = \theta_{x2} - \theta_{x1}$$

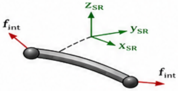
This is the heart: everything expressed in a frame where deformation is small—even if motion is not.

The corotational formulation doesn't fight nonlinearity— it reframes it.—
It separates what is rigid from what is deformable, so we can use complex motion.

5 INTERNAL FORCES

$$\{f_{int}\}_D = g(\{d_D\})$$

Computed in the corotational frame using stability functions or constitutive laws.



Back to the global system:

$$\{f_{int}\}^A = [R_{SR}] \cdot [H] \cdot \{f_{int}\}_D$$

6 CONSISTENT TANGENT STIFFNESS

$$\frac{d\{f_{int}\}^A}{d\{d_T\}} =$$

$$\left(\frac{d[R_{SR}]}{d\{d_T\}}\right) \cdot [H] \cdot \{f_{intD}\}$$

Term 1: rotation-induced geometry

$$[R_{SR}] \cdot \left(\frac{d[H]}{d\{d_T\}}\right) \cdot \{f_{intD}\}$$

Term 2: length evolution (often neglected!)

$$[R_{SR}] \cdot [H] \cdot \left(\frac{d\{f_{intD}\}}{d\{d_D\}}\right) \cdot \left(\frac{d\{d_D\}}{d\{d_T\}}\right)$$

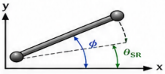
Term 3: material response

That second term is not decorative—
it is the difference between linear convergence and quadratic convergence in Newton–Raphson.

7 2D REDUCTION

$$[R_T] = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

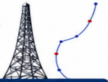
$$\theta_{pure} = \phi - \theta_{SR}$$



Same philosophy. Fewer dimensions. Same truth.

WHY THIS MATTERS

- Capture large rotations without nonlinear strain measures
- Preserve small-strain constitutive models
- Achieve robust and fast convergence
- Accurately predict buckling and post-buckling paths



In short: it lets structures rotate wildly while mathematics stays calm.