RESPONSE SPECTRUM METHOD
MODAL ANALYSIS – EQUATIONS SUMMARY

1 VIBRATION MODES (EIGENVALUE PROBLEM)

$$(\boldsymbol{K} - \omega_j^2 \boldsymbol{M})\{\boldsymbol{\phi}_j\} = \{0\}$$

- $\boldsymbol{K}$: global stiffness matrix
- $\boldsymbol{M}$: global mass matrix
- ω_j : circular natural frequency of mode j (rad/s)
- $\{\boldsymbol{\phi}_j\}$: mode shape (eigenvector) for mode j
- $j = 1, 2, \cdots, n$: mode number

n = number of degrees of freedom

2 PARTICIPATION FACTOR (MODAL PARTICIPATION)

$$PF_j = \frac{\{\boldsymbol{\phi}_j\}^T \boldsymbol{M}\{\boldsymbol{r}\}}{\{\boldsymbol{\phi}_j\}^T \boldsymbol{M}\{\boldsymbol{\phi}_j\}}$$

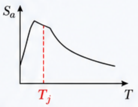
$\{\boldsymbol{r}\}$: participation vector
- 1 in dof related to the seismic excitation
- 0 in others

3 EQUIVALENT EXTERNAL FORCES FOR MODE j

$$\{\boldsymbol{f}_{ext}\} = PF_j\ S(T_j)\ 9.81\ \{\boldsymbol{\phi}_j\}$$

- $S(T_j)$: spectral acceleration corresponding to the period $T_j = \dfrac{2\pi}{\omega_j}$ (from the acceleration response spectrum)
- 9.81 : gravity acceleration (m/s^2)

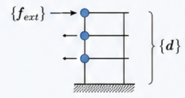
RESPONSE SPECTRUM



4 DISPLACEMENTS FOR EACH MODE j

$$\boldsymbol{K}\{\boldsymbol{d}\} = \{\boldsymbol{f}_{ext}\}$$

$\{\boldsymbol{d}\}$: displacement vector for mode j (response due to $\{\boldsymbol{f}_{ext}\}$)



5 COMBINATION OF MODES – SRSS

For node j and combining n modes:

Displacements:

$$d_j = \sqrt{\sum_{i=1}^{n} d_{ij}^2}$$

Shear forces:

$$V_{zj} = \sqrt{\sum_{i=1}^{n} V_{zij}^2}$$

- d_{ij} : displacement at node j due to mode i
- V_{zij} : shear force at level z due to mode i

6 EFFECTIVE MASS OF MODE j

$$mass_j = \frac{Base\ shear\ (j)}{S(T_j)\ 9.81} = (PF_j)^2\ \left(\{\boldsymbol{\phi}_j\}^T \boldsymbol{M}\{\boldsymbol{\phi}_j\}\right)$$

- $Base\ shear\ (j)$: total base shear obtained from mode j using $S(T_j)$
- $(PF_j)^2\ \left(\{\boldsymbol{\phi}_j\}^T \boldsymbol{M}\{\boldsymbol{\phi}_j\}\right)$: effective modal mass

7 MASS PERCENTAGE FOR MODE j

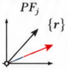
$$mass\%_j = \frac{mass_j}{Total\ mass} \times 100\ \%$$

- $mass_j$: effective mass of mode j
- $Total\ mass$: total mass of the structure ($= \sum$ of all translational masses in direction of excitation)

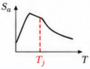
8 SUMMARY OF PROCEDURE

1. Compute modes $(\omega_j, \{\phi_j\})$

2. Compute participation factors PF_j



3. Get $S(T_j)$ and compute $\{f_{ext}\}$



4. Solve for $\{d\}_j$

5. Combine modes (SRSS)

$$\sqrt{\sum (\cdot)^2}$$

6. Compute base shear, effective mass and mass %