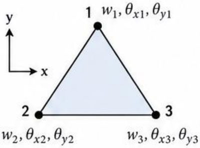
1. PLANE STRESS PROBLEM WITH BCIZ (TRIANGLE ELEMENT – KIRCHHOFF)



- **TRIANGLE SHAPE FUNCTIONS** N_i $(i = 1, \ldots, 9)$

$$N_i = a_i + b_i x + c_i y + d_i xy + e_i x^2 + f_i y^2$$

(total of 9 functions for the 9 DOFs)

- **DISPLACEMENTS**

$$w = w_1 N_1 + \theta_{x1} N_2 + \theta_{y1} N_3 + w_2 N_4 + \theta_{x2} N_5 + \theta_{y2} N_6 + w_3 N_7 + \theta_{x3} N_8 + \theta_{y3} N_9$$

- **STRAINS** (Kirchhoff plate)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -z \dfrac{d^2 w}{dx^2} \\ -z \dfrac{d^2 w}{dy^2} \\ -z \dfrac{d^2 w}{dxdy} \end{Bmatrix}$$

- **NORMAL AND SHEAR STRESSES (PLANE STRESS)**

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \dfrac{1-\mu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

- **EQUILIBRIUM (VIRTUAL WORK PRINCIPLE)**

$$\iiint (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy})' \, dV = \sum_{i=1}^{3} (F_{zi}\, \delta w_i + M_{xi}\, \delta\theta_{xi} + M_{yi}\, \delta\theta_{yi}) + \int_S (q_z\, \delta w + m_x\, \delta\theta_x + m_y\, \delta\theta_y)\, dS$$

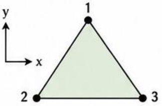
F_{zi} : transverse forces (z)

M_{xi}, M_{yi} : bending/twisting moments

q_z : transverse load (per area)

m_x, m_y : moment loads (per area)

2. PLANE STRESS PROBLEM WITH C.S.T. (CONSTANT STRAIN TRIANGLE)



- **TRIANGLE SHAPE FUNCTIONS**

$$N_i = a_i + b_i x + c_i y \quad (i = 1, 2, 3)$$

(linear functions)

- **DISPLACEMENTS**

$$\begin{Bmatrix} U \\ V \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \end{Bmatrix}$$

- **STRAINS** (constant)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & 0 & \dfrac{\partial N_2}{\partial x} & 0 & \dfrac{\partial N_3}{\partial x} & 0 \\ 0 & \dfrac{\partial N_1}{\partial y} & 0 & \dfrac{\partial N_2}{\partial y} & 0 & \dfrac{\partial N_3}{\partial y} \\ \dfrac{\partial N_1}{\partial y} & \dfrac{\partial N_1}{\partial x} & \dfrac{\partial N_2}{\partial y} & \dfrac{\partial N_2}{\partial x} & \dfrac{\partial N_3}{\partial y} & \dfrac{\partial N_3}{\partial x} \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \end{Bmatrix}$$

- **NORMAL AND SHEAR STRESSES (PLANE STRESS)**

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \dfrac{1-\mu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

- **EQUILIBRIUM (VIRTUAL WORK PRINCIPLE)**

$$\iiint (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy})\, dV = \sum_{i=1}^{3} (F_{xi}\, \delta u_i + F_{yi}\, \delta v_i) + \int_S (q_x\, \delta u + q_y\, \delta v)\, dS$$

F_{xi}, F_{yi} : nodal forces in x and y

q_x, q_y : surface loads (per area) in x and y

u, v : in-plane displacements