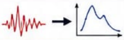
GENERATION OF A RESPONSE SPECTRUM FROM AN SDOF SYSTEM

A Precise Engineering Framework

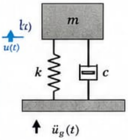
A response spectrum is not just a curve—it is a mapping from structural dynamics to extrema.
It compresses an entire time history into a set of peak responses of linear oscillators, each tuned to a different natural period.



1. GOVERNING EQUATION — THE DYNAMIC KERNEL

For a linear SDOF system subjected to base excitation:

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$



- Mass m → invariant (normalization)
- Damping ratio ξ → fixed (2–5%)
- Stiffness $k = m\omega_n^2 = m\left(\frac{2\pi}{T}\right)^2$

→ control variable

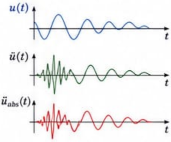
Sweeping the period T is equivalent to traversing stiffness space.

This is the only degree of freedom we allow to vary. Everything else is discipline.

2. RESPONSE QUANTITIES — WHAT WE ACTUALLY MEASURE

Solving the equation yields:

- Relative displacement: $u(t)$
- Relative velocity: $\dot{u}(t)$
- Absolute acceleration: $\ddot{u}_{abs}(t) = \ddot{u}(t) + \ddot{u}_g(t)$



We do not care about the full trajectories.
We extract only the extrema:

$S_d(T,\xi) = \max\lvert u(t)\rvert$ (Relative Displacement) → deformation demand	$S_v(T,\xi) = \max\lvert\dot{u}(t)\rvert$ (Relative Velocity) → energy rate	$S_a(T,\xi) = \max\lvert\ddot{u}_{abs}(t)\rvert$ (Absolute Acceleration) → inertial force

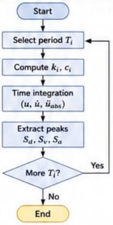
These correspond to deformation demand, energy rate, and inertial force.

3. THE ALGORITHM — INDUSTRIAL REALITY, NOT THEORY

1. For each period T_i

$$k_i = m\left(\frac{2\pi}{T_i}\right)^2$$
$$c_i = 2\xi m\omega_{n,i}$$

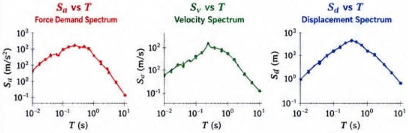
2. Integrate in time using:
   - Newmark-β (industry standard)
   - Wilson-θ (for robustness)
   - or exact convolution (Duhamel) if you want elegance over speed
3. Extract peak values → (S_d, S_v, S_a)
4. Move to next T_i



Repeat this hundreds of times. No shortcuts, no approximations.

4. SPECTRUM CONSTRUCTION — FROM TIME TO GEOMETRY

Plot the peak responses versus period T (log scale):



SHORT PERIODS ($T \to 0$)	INTERMEDIATE PERIODS	LONG PERIODS ($T \to \infty$)
System is stiff → follows ground $S_a \approx$ PGA Inertia dominates	Resonance amplification Peak structural demand lives here	System drifts with ground $S_d \approx$ PGD Kinematics dominates

← Period T (log scale) →

5. PSEUDO vs TRUE SPECTRA — WHERE ENGINEERS CUT CORNERS

PSEUDO (Design Practice)	TRUE (From Time History)
$S_{pa} = \omega_n^2 S_d$	$S_a = \max\lvert\ddot{u}_{abs}(t)\rvert$
$S_{pv} = \omega_n S_d$	$S_v = \max\lvert\dot{u}(t)\rvert$
	$S_d = \max\lvert u(t)\rvert$

Assumes harmonic behavior.
Real seismic input is not harmonic.

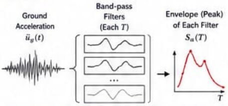
Translation: Useful for design (codes).
Not exact for physics.

An expert knows when that distinction matters:

- Linear analysis → acceptable
- Nonlinear / time-history calibration → dangerous simplification

6. PHYSICAL INTERPRETATION — WHAT THE SPECTRUM REALLY SAYS

The response spectrum is a **filter bank.**

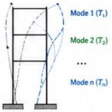


Each oscillator extracts energy from the ground motion at its natural frequency. The peak response reflects how well the input excites that frequency band.

In signal-processing terms:
You are computing the envelope of band-pass filtered responses.

7. STRUCTURAL APPLICATION — FROM SDOF TO REALITY

1. Perform modal decomposition
2. Assign each mode a period T_n (and damping ξ_n)
3. Read $S_a(T_n, \xi_n)$ from spectrum
4. Convert to modal forces $F_n = S_a(T_n, \xi_n) \cdot m_n \cdot \phi_n$
5. Combine modal responses (SRSS, CQC, or other method)



That is where the spectrum stops being academic and starts carrying design loads.

8. CRITICAL OBSERVATIONS (NO FLUFF)

- The spectrum is not unique → depends on the accelerogram.
- It is not invariant → damping reshapes it significantly.
- It is not predictive → it summarizes, it does not generalize.
- It is not plastic → strictly linear-elastic framework.

It tells you how hard the earthquake can hit, not whether your structure survives.

That requires capacity, ductility, and hierarchy—another battlefield entirely.

9. MINIMAL NUMERICAL ANCHOR

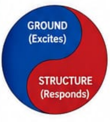
Example:
$m = 1000$ kg
$\xi = 5\%$
$T = 0.5$ s

→ **Compute:**
$\omega_n = \frac{2\pi}{T} = 12.57$ rad/s
$k = m\omega_n^2$
$\approx 1.58 \times 10^5$ N/m
$c = 2\xi m\,\omega_n$
$\approx 1.26 \times 10^4$ N·s/m

→ **Result:**
Same ground motion.
Different stiffness.
Entirely different fate.

That is the essence of the spectrum.

CLOSING INSIGHT



A response spectrum is a silent conversation between الأرض (the ground) and structure.

- Each point is a maximum.
- Each maximum is a warning.
- Each warning is period-dependent.

Change the period — and you change the story.