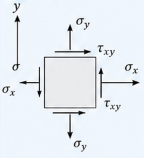
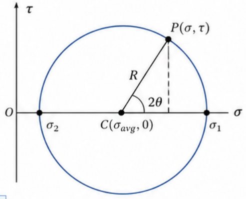
MOHR'S CIRCLE FOR STRESSES AND STRAIN ROSETTE

1. MOHR'S CIRCLE FOR STRESSES (PLANE STRESS)

Stress element





Equations of Mohr's circle

- **Center:**

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

- **Radius:**

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- **Equation of circle:**

$$(\sigma - \sigma_{avg})^2 + \tau^2 = R^2$$

Stresses on a plane making an angle θ (counterclockwise from the x-face):

$$\sigma_\theta = \sigma_{avg} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal stresses (at $\tau_\theta = 0$):

$$\sigma_{1,2} = \sigma_{avg} \pm R$$

Key points

- σ_1: maximum principal stress (rightmost point)
- σ_2: minimum principal stress (leftmost point)
- Maximum in-plane shear stress: $\tau_{max} = R$ (at top/bottom of circle)
- Orientation of principal planes:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

3D stress transformation (for a unit normal vector $\boldsymbol{n} = \{n_x, n_y, n_z\}$)

Normal stress:

$$\sigma_n = \boldsymbol{n}^T \boldsymbol{\sigma} \boldsymbol{n} = (n_x \quad n_y \quad n_z) \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

Shear stress vector:

$$\boldsymbol{\tau} = \boldsymbol{\sigma n} - \sigma_n \boldsymbol{n}$$

with

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Hooke's Law (3D, isotropic, linear elastic)

Shear:

$$\tau = G\gamma$$

Normal:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix}$$

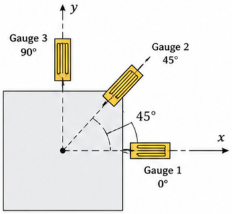
where

$$G = \frac{E}{2(1+\nu)}$$

(E: Young's modulus, ν: Poisson's ratio)

2. STRAIN ROSETTE (0°–45°–90°)

A strain rosette uses three strain gauges to measure normal strains in different directions at a point. The most common is the **0°–45°–90° rectangular rosette.**



Strain transformation

Normal strain in a direction θ (measured counterclockwise from the x-axis):

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

For a 0°–45°–90° rosette (measurements: $\varepsilon_0, \varepsilon_{45}, \varepsilon_{90}$)

$$\varepsilon_x = \varepsilon_0$$

$$\varepsilon_y = \varepsilon_{90}$$

$$\gamma_{xy} = 2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90}$$

Principal strains and orientation

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

θ_p is the angle from the 0° direction to the major principal strain ε_1 (counterclockwise).

SUMMARY

- Mohr's circle provides a graphical method to find normal and shear stresses on any plane in a 2D stress state.
- The strain rosette allows determination of in-plane strains ($\varepsilon_x, \varepsilon_y, \gamma_{xy}$) and principal strains from three measured normal strains.
- These tools are fundamental for stress analysis, failure theories, and structural design.