Abstract

This article presents a conceptual framework for understanding problem-solving as a process of transformation across domains. Drawing from mathematics, physics, and the arts, it proposes a four-stage structure: (1) Problem Definition through Initial State and Evolution Laws, (2) Transformation to a different domain (B), (3) Resolution within that domain, and (4) Inverse Transformation back to the original context. The inclusion of artistic domains - often dismissed as subjective - is central, positioning them as cognitive spaces for insight and metaphorical reasoning. Examples range from mathematical substitutions and Feynman diagrams to fractals, algorithmic art, and performance as analysis.

1. Introduction

Problem-solving is a fundamental human activity that spans disciplines - from engineering and physics to literature and visual arts. While conventional approaches often emphasize logic and formalism, many real-world problems remain resistant to such reduction. What if the essence of problem-solving lies not in direct attack, but in transformation - a shift in domain that opens new affordances for perception and resolution?

This article proposes a general theory: problem-solving as domain transformation. It integrates mathematical rigor with artistic intuition, allowing analytical and aesthetic reasoning to coexist as part of the same epistemological loop.

2. The Four-Part Framework

2.1 Definition - Initial State and Evolution Laws

Every problem begins with an initial configuration: a set of conditions or variables (S), and a set of rules or laws (L) that determine how those conditions evolve. This model echoes Laplacian determinism - in theory, perfect knowledge of S and L yields full predictability. But real problems

often involve incomplete or ambiguous laws and hidden variables. Hence, defining a problem is an act of interpretive and creative constraint.

Key processes in definition:

- Observation: What can we perceive?
- Selection: What is essential?
- Framing: What governs change?

This reframes problem definition as a cognitive modeling process - not a neutral act, but one shaped by intention and metaphor.

2.2 Transformation - Moving Into Domain B

When a problem is hard to manage in its original domain (A), we apply a transformation T that maps it into a new domain (B) where it becomes tractable, expressive, or perceptible in a different way. Domain B could be:

- **Analytical** (e.g., variable change in calculus, Laplace/Fourier transform)
- **Visual** (e.g., diagrams, graphs, fractals)
- **Auditory** (e.g., sonification)
- **Artistic** (e.g., painting, poetry, performance)

Transformation is more than simplification - it's re-visioning. It allows form, rhythm, and metaphor to become analytical tools.

2.3 Resolution in Domain B

Once transformed, the problem is engaged with via the logic, structure, or affective language of domain B. This includes:

- **Variable substitution**: Turning xe^(x) dx into a solvable integral via u = x.
- **Fourier transform**: Decomposing signals into frequency components.
- **Fractals**: Recasting complexity through self-similar visual structures.
- **Feynman diagrams**: Visual metaphors for quantum interactions.
- **Artistic representation**: A social issue rendered in dance, making relational tensions visible through movement.

In each case, the domain enables new operations, perceptions, or gestures of interpretation.

2.4 Inverse Transformation - Returning to Domain A

After resolution, the insight is mapped back into the original domain. The goal is not merely translation, but often revelation. The process reshapes the original frame, leading to:

- New models
- Altered questions
- Unanticipated resolutions
- Emergent understanding

3. Examples in Detail

3.1 Variable Substitution as Domain Shift

In calculus, variable substitution (u-substitution) is a formal domain change. By reframing x in terms of u = g(x), we transfer the problem to a space where it is solvable. This mirrors the broader idea of

cognitive reframing: problems are often resolved not by working harder in A, but by moving to B.

3.2 Feynman Diagrams

A quantum interaction, initially encoded in symbolic equations, is transformed into a spatial-temporal metaphor. These diagrams permit physicists to "see" and estimate complex processes, blending visual and mathematical reasoning.

3.3 Fractals and Visual Reframing

The Mandelbrot set transforms complex number iteration into a visual language, revealing infinite self-similarity and bifurcating structures. This is a domain shift from algebra to topology and perception.

3.4 Artistic Transformations

- **Xenakis**: Used stochastic physics to compose music, turning mathematical randomness into sonic tension.
- **Mozart**: Structured compositions with both emotional arc and mathematical symmetry - domain blending at its finest.
- **Harold Cohen**: Created algorithmic visual art through rule-based systems.
- **Performance Art**: Representing abstract societal conflict through embodied metaphor.

In all cases, Domain B (art) is not an aesthetic afterthought but a cognitive engine.

4. Implications and Applications

This framework has wide applicability:

- **Education**: Teaching students to change representations builds cognitive flexibility and creativity.
- **AI & Cognition**: Models of intelligent behavior must account for multimodal, metaphorical problem-solving.
- **Interdisciplinary Research**: Domain transformation is at the heart of innovation across fields.
- **Humanities**: Offers a rigorous model for how metaphor, narrative, and symbol act as tools of analysis.

5. Conclusion

To solve a problem is to transform it. Whether through a change of variables, a change of medium, or a change of mind, we move between cognitive domains to find clarity, structure, or resonance. Artistic modes are not peripheral - they are epistemic engines. This theory reframes thinking itself as a dynamic loop: Definition Transformation Resolution Inversion New World.

A better question is not "What is the answer?" but "In which domain does the answer become visible?"